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* In Kazan University the Electron Paramagnetic Resonance (EPR) was discovered by Zavoisky E.K. in 1944.

Local magnetic field distribution of the vortex lattice near surface of superconducting plate[†]

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Within the framework of the London model the distribution of local magnetic field near the surface of plate of anisotropic type-II superconductor is found when the external magnetic field is perpendicular to the axis of symmetry of the crystal. There is obtained the distribution of the local magnetic field depending on the distance to the surface of the superconducting plate. It is shown that the lineshape of distribution of local magnetic field near the surface changes considerably as compared with the distribution in the depth of massive superconductor. This change should be taken into account when interpreting experimental data on the observation of the local magnetic field in the near-surface region of massive superconductor and in thin superconducting films (thickness is less than the depth of penetration of the magnetic field in the superconductor).

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1. Introduction

The methods of nuclear magnetic resonance (NMR) is now widely used to study the properties of type-II superconductors including high-temperature ones. NMR experiments have played a big role in understanding of the dynamics of vortices in high-temperature superconductors. For a reliable interpretation of lineshape along with the uniform width determined by the nonhomogeneous dynamics of interaction of the spin system of nuclei with the conduction electrons and between them, it is necessary to take into account the inhomogeneity of the local magnetic field in the superconductor, i.e. the inhomogeneous NMR line width [1]. In the study of the vortex lattice by the NMR method the distribution of magnetic field is usually used which is formed in the thickness of the massive superconductor, assuming that the inhomogeneity of the local field is the same both in the depth of the superconductor and on its surface. However, it is known [2–4] that the local magnetic field considerably changes as it approaches the surface of the superconductor. However, the spatial distribution of the magnetic field in the superconductor near its surface significantly differs from the distribution of the local field in the depth of the superconductor. In the present paper this problem is solved on the basis of the generalized London equations with the use of appropriate boundary conditions. There has been found the dependence of distribution of magnetic field in the unit cell of the vortex lattice on the distance to the surface of superconducting plate when the external magnetic field is perpendicularly directed to the axis of symmetry of the crystal. Numerical calculations showed that for intermediate values of magnetic field $H_{c1} < H < H_{c2}$ the distribution of fields of vortex lattice significantly changes near the surface and turns into a uniform field over the surface at

[†]This paper is originally written by authors on the occasion of eightieth birthday of Professor Boris I. Kochelaev.

the distances of the order of the averaged penetration depth of the magnetic field. There have been obtained map of magnetic field distribution,the lineshape of nuclear magnetic resonance depending on the distance to the surface of the anisotropic superconductor.

2. Basic equations

We will consider a superconducting plate of thickness d placed in a magnetic field and occupying the space $-d/2 < z < d/2$. The density of free energy of an anisotropic superconductor [1–3]:

$$8\pi F = \int \left(\mathbf{h}_s^2(\mathbf{r}) + \lambda^2 m_{\alpha\beta}\,\mathrm{curl}_\alpha\,\mathbf{h}_s(\mathbf{r})\,\mathrm{curl}_\beta\,\mathbf{h}_s(\mathbf{r})\right) dv, \tag{1}$$

where $\mathbf{h}_s(\mathbf{r})$ is the local magnetic field in the superconductor, dv is a volume element; $m_{\alpha\beta} = M_{\alpha\beta}/M_{av}$ is a given phenomenological tensor; $M_{av} = (M_1M_2M_3)^{1/3}$ and M_{av} are the principal values of the "mass tensor" $M_{\alpha\beta}$. The principal values of mass tensor $m_{\alpha\beta}$ are related by the relation $m_1m_2m_3 = 1$. For the majority of high-temperature superconductors it is possible to put $m_1 = m_2 < m_3$ and then it is only needed to know the mass ratio $\Gamma = m_3/m_1$. The anisotropic parameter Γ can be determined experimentally [1, 5], for example, from the ratio of two critical fields directed in one case along the plane ab, and the other along the crystal axis c. In the system of coordinates (X, Y, Z) related to the basic crystal axes where the Z axis is perpendicular to the layers the tensor $m_{\alpha\beta}$ is diagonal [1]. Varying the formula for the free energy of the anisotropic superconductor (1) with respect to $\mathbf{h}_s(\mathbf{r})$, we will get the following equations for the magnetic field:

$$h_{s,\alpha} - \lambda^2 m_{kl}\varepsilon_{\alpha l\delta}\varepsilon_{k\gamma\beta}h_{s\beta;\delta\gamma} = \Phi_0\nu_\alpha \sum_\nu \delta(\mathbf{r} - \mathbf{r}_\nu). \tag{2}$$

Here the notation is introduced, for example, $h_{s\beta;\delta\gamma} = \partial^2 h_{s\beta}/\partial\delta\partial\gamma$, where $\alpha, \beta, \gamma, \delta = x, y, z$; further, Φ_0 is the magnetic flux quantum, so that the magnetic induction $B = \Phi_0/S$, S is the area of the unit cell of the vortex lattice, $\varepsilon_{\alpha l\delta}$ is the antisymmetric tensor, $\boldsymbol{\nu}$ is the unit vector along the direction of vortices. In the last equation the term in the right-hand side takes into account the singularities in the vortices.

In the region $z < d/2$ and $z > d/2$ the distribution of the magnetic field is defined by the Maxwell equations in vacuum:

$$\begin{aligned} &\mathrm{curl}\,\mathbf{h}_v(\mathbf{r}) = 0,\\ &\mathrm{div}\,\mathbf{h}_v(\mathbf{r}) = 0, \end{aligned} \tag{3}$$

where $\mathbf{h}_v(\mathbf{r})$ is the magnetic field in the vacuum.

The equations (1) and (2) on the boundary between two media are supplemented with the following boundary conditions:

$$\begin{aligned} &h_{sn}(x, y, \pm d/2) = h_{vn}(x, y, \pm d/2),\\ &(\mathrm{curl}\,\mathbf{h}_v(\mathbf{r}))_z|_{z=0} = 0, \end{aligned} \tag{4}$$

Here $h_{sn}(\mathbf{r})$ and $h_{vn}(\mathbf{r})$ are the normal components of the field in the superconductor and in the vacuum, respectively. From the physical point of view the first condition means that the normal components of the field on the boundary of the media change continuously, and the second condition takes into account that fact that the component of the current normal to the surface is absent.

The field $\mathbf{h}(\mathbf{r})$ has the periodicity of the vortex lattice in the plane (x, y), so that we can expand it in a Fourier series:

$$\begin{aligned} \mathbf{h}(\boldsymbol{\rho}, z) &= \sum_{\mathbf{G}} \mathbf{f}(\mathbf{G}, z) e^{i\mathbf{G}\boldsymbol{\rho}}, \\ \mathbf{f}(\mathbf{G}, z) &= \frac{B}{\Phi_0} \int \mathbf{h}(\boldsymbol{\rho}, z) e^{-i\mathbf{G}\boldsymbol{\rho}} d\boldsymbol{\rho}, \end{aligned} \tag{5}$$

where $\mathbf{h}(\boldsymbol{\rho}, z)$ is the Fourier transform of magnetic field, $\mathbf{G}$ is the reciprocal lattice vector, $\boldsymbol{\rho}$ is the position vector. The solutions for the Fourier components of the field satisfying the boundary conditions (4) when $\mathbf{G} \neq 0$ have the form:

in the superconductor $(-d/2 \leq z \leq d/2)$

$$\begin{aligned} f_{sx}(G, z) &= -i\frac{BG_x(1 + \lambda^2 m_1 G_x^2)}{q_3 D_2} \sinh\left(\frac{q_3 d}{2}\right) \sinh(q_1 z), \\ f_{sy}(G, z) &= -i\frac{BG_y}{q_3 D_2} \left\{\sinh\left(\frac{q_1 d}{2}\right) \sinh(q_3 z) + \lambda^2 m_1 G_x^2 \sinh\left(\frac{q_3 d}{2}\right) \sinh(q_1 z)\right\}, \end{aligned} \tag{6}$$

$$\begin{aligned} f_{sz}(G, z) = \frac{B}{\lambda^2 m_3 q_3^2} - \frac{B}{q_3 D_2} \Bigg\{ & G_y^2 \sinh\left(\frac{q_1 d}{2}\right) \cosh(q_3 z) \\ & + \lambda^2 m_1 q_1 q_3 G_x^2 \sinh\left(\frac{q_3 d}{2}\right) \cosh(q_1 z) \Bigg\}, \end{aligned}$$

in the vacuum $(|z| > d/2)$

$$\begin{aligned} f_{vx}(G, z) &= \pm i\frac{BG_x(1 + \lambda^2 m_1 G_x^2)}{q_3 D_2} \sinh\left(\frac{q_1 d}{2}\right) \sinh\left(\frac{q_3 d}{2}\right) e^{G(\frac{d}{2} \pm z)}, \\ f_{vy}(G, z) &= \pm i\frac{BG_y(1 + \lambda^2 m_1 G_x^2)}{q_3 D_2} \sinh\left(\frac{q_1 d}{2}\right) \sinh\left(\frac{q_3 d}{2}\right) e^{G(\frac{d}{2} \pm z)}, \\ f_{vz}(G, z) &= \pm i\frac{BG(1 + \lambda^2 m_1 G_x^2)}{q_3 D_2} \sinh\left(\frac{q_1 d}{2}\right) \sinh\left(\frac{q_3 d}{2}\right) e^{G(\frac{d}{2} \pm z)}, \end{aligned} \tag{7}$$

where

$$q_1 = \frac{1}{\lambda^2 m_1} + G^2, \qquad q_3 = \frac{1}{\lambda^2 m_1} - \frac{m_1}{m_3} G_x^2 + G_y^2, \qquad G = \sqrt{G_x^2 + G_y^2},$$

$$\begin{aligned} D_2 = \lambda^2 m_3 \Bigg\{ & G_y^2 \sinh\left(\frac{q_1 d}{2}\right) \cosh(q_3 z) + q_3 G(1 + m_1 G_x^2) \sinh\left(\frac{q_1 d}{2}\right) \\ & + \lambda^2 m_1 q_1 q_3 G_x^2 \cosh\left(\frac{q_1 d}{2}\right) \Bigg\} \sinh\left(\frac{q_3 d}{2}\right), \end{aligned}$$

the upper sign corresponds to the case when $z > d/2$ and the lower sign corresponds to the case when $z < -d/2$.

3. Distribution of magnetic field

Since it is difficult to analytically perform the inverse Fourier transform of the obtained solutions, then we will do it numerically. The pattern of distribution of the magnetic field in the real space is obtained by means of the standard procedure of fast fourier transform (FFT) for two-dimensional periodic functions. Following the papers [1–3], we introduce a nonorthogonal system of reference η, ζ:

$$\eta = x - y\cos\phi, \qquad \zeta = y/\sin\phi. \tag{8}$$

In this system of reference $\mathbf{h}(\eta+nb_1, \zeta+mb_2) = \mathbf{h}(\eta, \zeta)$, where m, n are integer numbers. Then, instead of (5) we have:

$$\begin{aligned} \mathbf{h}(\eta, \zeta, z) &= \sum_{n,m} \mathbf{f}(p, q, z) e^{i(p\eta+q\zeta)}, \\ \mathbf{f}(p, q, z) &= B \int d\eta\, d\zeta\, \mathbf{h}(\eta, \zeta, z) e^{-i(p\eta+q\zeta)}, \end{aligned} \tag{9}$$

where $p = \frac{2\pi n}{|\mathbf{b}_1|}$,$q = \frac{2\pi n}{|\mathbf{b}_2|}$, $\mathbf{b}_1$ and $\mathbf{b}_2$ are basis vectors of unit cell of the vortex lattice.

It is convenient to introduce the dimensionless quantities $\mathbf{h} = \mathbf{h}\lambda^2/\Phi_0$, $\mathbf{B} = \mathbf{B}\lambda^2/\Phi_0$, $\boldsymbol{\rho} = \boldsymbol{\rho}/\lambda$, $\mathbf{G} = \mathbf{G}\lambda$ i.e. the length is measured in the units of length of the penetration depth of the magnetic field, and the field is measured in the units of Φ_0/λ^2. To present results we will use numerical estimates for the anisotropic parameter Γ and the field B given in the papers [1, 6]. We will choose the mass ratio $\Gamma = m_3/m_1 = 25$, which reflects the anisotropy of the high-temperature superconductor $YBa_2Cu_3O_{7-\delta}$ with $T_c = 90\,K$. We will take the magnetic field $h = 2$, which is much higher than H_{c_1} in the interval interesting for us $0 < \theta < \pi/2$.

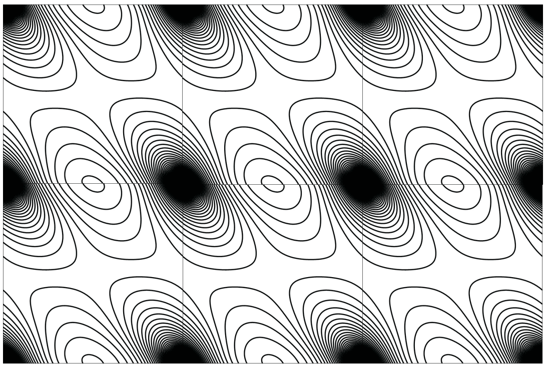

Figure 1. The map of the distribution of the longitudinal component of the magnetic field h_z on the surface of superconducting plate $(z = d/2)$ in the unit cell of the vortex lattice when $\mathbf{B}\perp\mathbf{c}$.

In the Fig. 1 the map of distribution of longitudinal component of magnetic field h_z in the unit cell of the vortex lattice is presented in the case when the external magnetic $\mathbf{B}$ is perpendicularly directed relative to the axis of symmetry of the crystal on the surface of superconducting plate $(z = d/2)$. The distance to the surface is given in the units of the averaged depth of penetration of the magnetic field, λ; the magnetic field is in the units of Φ_0/λ^2. It should be noted that the distribution of the fields of the vortex lattice significantly changes near the surface of the superconductor and turns into the uniform one over the surface at distances of the order of the averaged depth of penetration of the magnetic field λ.

4. Lineshape of magnetic resonance

The inhomogeneous distribution of the magnetic field $|\mathbf{h}|$ in the unit cell of the vortex lattice, obviously, makes a contribution to the broadening of the magnetic resonance line in the superconductor [4, 7, 8]. In the case when it is the only source of broadening the form of the magnetic resonance line, in particular NMR accurately reproduces the distribution function of the local fields in the vortex lattice [4, 8]. In order to get a more detailed picture of the distribution of the fields as a function of distance to the surface of the superconductor, we will calculate the lineshape of the magnetic resonance for the layer at a distance z (in the units of λ) above the surface of the superconducting plate. Meanwhile the accuracy of calculation is determined by the number of points into which the unit cell of the vortex lattice is divided in which the local magnetic field $\mathbf{h}(\mathbf{r})$ is calculated (in our case 1024×1024 points were used). The form of the resonance line is determined by direct count of the relative number of points in the plane (x, y) for which the value of the field lies in h and $h + \delta h$, while the wings of the magnetic resonance line correspond to the maximum of the magnetic field located at the center of the vortex and the minimum in the valley of relief of the field distribution, and the peak of the NMR line corresponds to the saddle points which are on lines, connecting tops. The results of such calculation are shown in Fig. 2. In this figure the values of the field h are plotted on the axis of abscissa, from $h_{\min}$ in the middle of the superconducting plate ($z = 0$) to B, i.e., part of curves for $h > B$ is not shown (in this region they fall monotonically to zero). The values of the minimum field $h_{\min} = 1.9778$. As expected, the magnetic resonance line narrows and its peak shifts to the value of the external field as we move from the depth of the superconductor to the surface and beyond it.

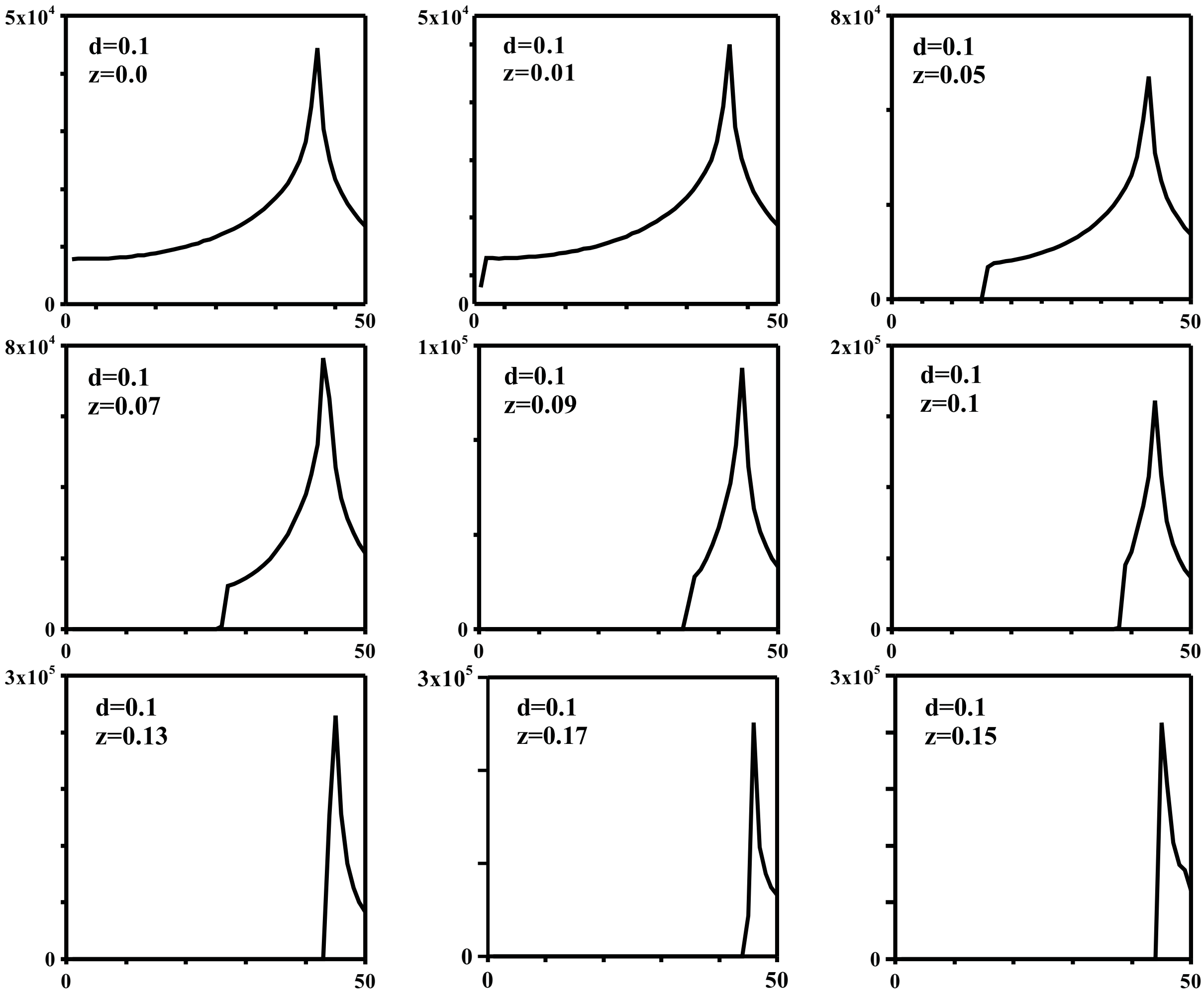

Figure 2. The line form of magnetic resonance at $B = 2.0$ depending on the distance to the surface of thin superconducting plate with the thickness $d = 0.1$. The values of the field $50(h - h_{\min})/(B - h_{\min})$ are plotted on the abscissa axes, the intensity in relative units is plotted on the ordinate axes.

5. Conclusions

The performed calculations showed that the distribution of the local magnetic field of the vortex lattice near the surface of the superconductor still has the features of the vortex lattice, but is very different from the field in the thickness of the superconductor and becomes uniform over the surface at distances from it of the order of the averaged penetration depth of the magnetic field. A characteristic feature of the local field near the surface is the presence of transverse field components, which are different from zero for any orientation of the external magnetic field relative to the crystal axes, while in the thickness of the superconductor they appear only in oblique orientations of the field. It should be noted that in the study of the vortex lattice by

NMR methods with the help of a probe on the surface of the superconductor, there appears an additional broadening of the resonance line, if the thickness of the probe is not negligible as compared with the magnetic field penetration depth λ. This is due to the fact that the position of the peak of the NMR line depends on the distance to the surface of the superconductor, and the resulting line shape is determined by the superposition of the resonance lines shown in Fig. 2. The stated shows that any determination of magnetic parameters of superconductor associated with the measurements of the local field near its surface should be based on the appropriate analysis of the near-surface distribution of the fields of the vortex lattice.

References

1. Thiemman S. L., Radovic Z., Kogan V. G., *Phys. Rev.* **39**, 11406 (1989).

2. Kochelaev B. I., Sharin E. P., *Superconductivity: Physics, Chemistry, Technics* **5**, 1982 (1992), [in Russian].

3. Kochelaev B. I., Sharin E. P., *Superconductivity: Physics, Chemistry, Technics* **5**, 1931 (1992), [in Russian].

4. Pozek M., Habermeier H. U., Maier A., Mehring M., *Physica C* **269**, 61 (1996).

5. Kogan V. G., Simonov A. Y., Ledvij M., *Phys. Rev. B* **48**, 392 (1993).

6. Farrell D. E., Williams C. M., Bannal N. P., Kogan V. G., *Phys. Rev. Lett.* **61**, 2805 (1988).

7. Brandt E. H., *Phys. Rev. B* **37**, 2349 (1988).

8. Maclaughlin D. E., *Solid State Phys.* **31**, 1 (1976).