# NMR Line Shape in Anisotropic Superconductors in Inclined Magnetic Fields.

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#### Abstract

The nuclear magnetic resonance line shapes within a primitive cell of the vortex lattice of the type II anisotropic superconductors in inclined magnetic fields are constructed taking into account a change of the local magnetic field inhomogeneity and skin - effect near the superconductor surface. The results of the numerical calculations show that the change of the direction of an external magnetic field modifies the parameters of the nuclear magnetic resonance line shapes. The derivative of the power of the absorption energy with respect to the magnetic field is calculated. It allows to obtain more detailed information about parameters of the superconductor such as an anisotropy parameter.

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#### 1 Introduction.

The nuclear magnetic resonance (NMR) is well known as a useful tool for the investigation of high- $T_c$  superconductors properties. To interpretate the NMRline shape the following three important circumstances are necessary to take into account: the first, homogeneous width of the line, the second, inhomogenenuity of a local magnetic field  $h(\mathbf{r})$  in a superconductor, the third, peculiarities of a penetration of a microwave magnetic field in a superconductor. Because the variable electromagnetic field penetrates into a superconductor on depth by an order of the  $\lambda$  ( $\lambda$  - the penetration depth of a magnetic field in a superconductor) [1] only, it is necessary to take into account the inhomogenenuity of the magnetic field  $h(\mathbf{r})$  in a narrow area near the superconductor surface. However, the inhomogeneruity of the magnetic field in a vortex lattice near the surface of the type II superconductor considerably differs from the inhomogenenuity  $h(\mathbf{r})$  in a bulk superconductor [2]. In work [3] in the case of the perpendicular orientation of the external homogeneous magnetic field, the lineshape of the NMR line is constructed using a real change of the inhomogenenuity of the vortex lattice near the superconductor surface and was shown that these changes strongly effect on parameters of the NMR line. Besides, the surface effects drastically change the conclusions about the of the vortex lattice and parameters of the superconductor that usually are taken from the analysis of the NMR lineshape [4]

As High temperature superconductors are strongly anisotropic it is interesting to consider the changes of the NMR linehshapes parameters in the inclined external magnetic fields to the surface of the superconductor. In present work the NMR lineshape is considered in view of the surface effects that depend on angle  $\theta$  between normal to a surface of the superconductor and direction of the external magnetic field. It is shown that the lineshape essentially is modified for various  $\theta$ .

## 2 Calculation of a distribution function.

We consider the anisotropic type II superconductor occupying the half-space z < 0 in external magnetic field which is directed at angle  $\theta$  ( $\theta$  - the angle between vector H and axis z) assuming the axis z is parallel to axis c of the superconductor. If  $H_{c1} < H < H_{c2}$  ( $H_{c1}, H_{c2}$  - the low and upper critical fields respectively) the local magnetic field  $h(\mathbf{r})$  penetrate into the superconductor as quantum Abricosov vortices and becomes sharply inhomogeneous. It has also the short range order in the general case of the anisotropic vortex lattice and the lattice period is defined by value H. It is possible to expand the field  $h(\mathbf{r}, \theta)$  into Fourier series on reverse vectors  $\mathbf{G}$  of the vortex lattice. In work [5], the analytical expression for the Fourier-component of the local magnetic field as functions z and  $\theta - h_{\mathbf{G}}(z, \theta)$  is obtained on the basis of London equations

using the appropriate boundary conditions. The map of magnetic field  $h(\mathbf{r}, \theta)$  in sections z = const for the certain values of  $\theta$  can be restored by summarizing the Fourier- series using the values  $h_{\mathbf{G}}(z, \theta)$ .

The next step of the calculations is the determination of the distribution function  $\rho(h, z, \theta)$  of the local magnetic field  $h = |\mathbf{h}|$  in the elementary cell of the vortex lattice for the thin (in comparison to  $\lambda$ ) layer that stands on the distance z from the surface of the superconductor. The function  $\rho(h, z, \theta)$  is found by the calculation of the relative number of points in the plane (x, y) in the elementary cell of the vortex lattice for which the value of the local magnetic field is ranged between h and h + dh (dh = (H - hmin)/100, where H = 2 - value of the external magnetic field; the value of the minimal field in the superconductor  $h_{min} = 1.8767$  was defined on depth z = -5.0 (regime "bulk"). Two wings of the function will correspond the maximum of the magnetic field which is placed in the centre of the vortex, and the minimum in the valley of the relief of the distribution of the field. The peaks of the function correspond saddle points. For the analysis  $\rho(h, z, \theta)$  the maps of the distribution of the magnetic field for the different distances from the surface z and angle  $\theta$  are used. The elementary cell of the vortex lattice is partitioned into  $512 \times 512$  points at which  $h(\mathbf{r}, \theta)$  were calculated. For example, the functions  $\rho(h, z, \theta)$  are presented in Fig.1 for some values of z and  $\theta$ . Here and further dimensionless units are used : the distance is defined in terms of  $\lambda$  and the magnetic field - in terms of  $\Phi_0/\lambda^2$ , where  $\Phi_0$ - the quantum of the magnetic flux. The functions  $\rho(h, z, \theta)$  are calculated for the superconductor with the parameter of the anisotropy , = 25 ( ,  $= m_3/m_1$ ,  $m_1 = m_2, m_3$  - main meanings of "mass-tensor"). The parameter , = 25 reflects the anisotropy of the high-temperature superconductor Y - Ba - Cu - Owith  $T_c = 90K$ . The distribution function of the local magnetic field  $\rho(h, z, \theta)$ was calculated with step from  $\Delta z = 0.05$  from the surface of the superconductor to z = -0.7. Then the step was increased because on depth of the order  $\lambda/2$ the function  $\rho(h, z, \theta)$  approaches to the distribution function density of the magnetic field in the unlimited superconductor (curve "bulk") and practically not varies on the interval from  $z \leq -0.5$  to z = -5.0. As it is shown in Fig.1 the distribution function of the local magnetic field essentially depends on the distance from the surface of the superconductor but also is changed with angle  $\theta$ . In the case of the inclined external magnetic field there is additional peak in the  $\rho(h, z, \theta)$ . It is connected with the fact that in the relief of the distribution of the magnetic field the saddle points have different heights.

#### **3** Analysis of *NMR* lineshape.

Analysing the NMR lineshape we take into account that the electromagnetic microwave field penetrating into the superconductor is changed both on size and on phase. In type II superconductors due to the screening by superconducting currents an alternating field penetrates into the superconductor on depth  $\tilde{\lambda}$ 



Figure 1: distribution function of the local magnetic field  $\rho(h, z, \theta)$  in the elementary cell of the vortex lattice in the superconductor (in any units). The dashed line corresponds  $\theta = 0$ . The straight line is designed for the case  $\theta = \pi/6$ . For the mode "bulk" there corresponds z = -5.0. Along abscissa axis the meanings are drown of the field in terms of 100(h - hmin)/(H - hmin).

decreasing at amplitude. However, it phase is changed on the greater depthes [1]. As a result [6], for the NMR in the type II superconductor case, if homogeneous broadening  $\Delta$  is much less a spread of the local fields the absorbing microwave capacity is proportional to the imaginary part of the microwave susceptibility  $\chi$ " and lineshape is defined by the distribution of the local magnetic field. Let us calculate the power of the alternating magnetic field which is absorbed by resonant nuclear spins if they are located in a narrow layer z, z + dz. It is clear that it is proportional to  $\tilde{e}xp(2z/\delta)\rho(h,z)dz$ . The exponential multiplier takes into account that the amplitude of the alternating magnetic field is decreased exponentially with the removing from the superconductor surface. The  $\delta$  is equal to depth of the penetration of the alternating magnetic field with frequency  $\varpi$ in the superconductor. Let us consider that the homogeneous broadening is described by Lorentzian with width  $\Delta$ . The power of the alternating field which is absorbed by all resonant spins as a function of the external homogeneous magnetic field H for the isotropic g-factor can be written [3]:



Figure 2: Dependence of the absorption energy  $P(H,\theta)$  for  $\theta = \pi/6$ . The dashed line corresponds to the bulk mode without account of the surface effects. The straight line is depicted with taking into account the surface effects ( $\Delta = 1, \delta = 1$ ). The designation of the abscissa axis is the same as in Fig.1.

$$P(H,\theta) = C \int dh \cdot \frac{\Delta}{\Delta^2 + (H-h)^2} \int dz \cdot \rho(h,z,\theta) \exp\left(\frac{2z}{\delta}\right)$$
(1)

Here the constant C is defined by the normalization condition  $\int P(H,\theta)dH =$ 1. In Fig.2 the calculated NMR line is presented using the (1) ( the absorption energy P(H,q) for the anisotropic superconductor with parameters  $= 25, \Delta = 1, \delta = 1$  in the case when  $\theta = \pi/6$ .) For comparison the NMR lineshape of the superconductor with the same parameters but without taking into account changing of the inhomogeneous field with the removing from the surface of the superconductor, i.e. when  $\rho(h, z, \theta) = \rho(h, -\infty, \theta)$  (dashed line). As one can see the taking into account of the surface effects changes essentially the lineshape in the case of inclined fields as well as in the case when the magnetic field H is parallel z-axis. In Fig.3 the NMR lines are given for the different angles  $\theta$  from 0 to  $\pi/2$  with step  $\pi/12$ . There is a small displacement of the topfield peak and it is more pronounced for the low-field peak. After appearance of the deviation angle  $\theta$  the lineshape sharply changed its own characteristics. The additional peak is visible obviously and it is decreased with the increasing  $\theta$  and almost disappears at  $\theta = \pi/3$ . The lines of the absorption coincide practically at  $\theta = \pi/3$  and  $\theta = 5\pi/12$  (i.e. when  $\theta \to \pi/2$ ).



Figure 3: The dependence of the absorption for various meanings of angle  $\theta$  in view of the surface effects ( $\Delta = 1$ ,  $\delta = 1$ , mode skin):  $a - \theta = 0$ ;  $b - \theta = \pi/12$ ;  $c - \theta = \pi/6$ ,  $d - \theta = \pi/4$ ,  $e - \theta = \pi/3$  and  $\theta = 5\pi/12$ . The designations on x-axis meet in Fig.1.

The changes of the absorption features of the microwave energy are well appreciable on the lineshape of the derivative of absorption energy with respect to magnetic field dP/dH. It is shown in Fig.4 for the different angles and various meanings of  $\Delta$  with and without account of the changes of the local field near surface of the superconductor.  $\theta = \pi/6$ . Under increasing the parameter  $\Delta$  that describes homogeneous broadening two narrow low-field peaks are merged into one broaden and additional top-field wide peak is more seen on diagrams and it corresponds to the absorption energy of the magnetic field near surface. The main importance of all curves is that the parameter of the line asymmetry A/B(A/B- relation of the key low-field peak to the key top-field peak in the derivative of the absorption energy with respect to the magnetic field) grows approximately 2 times if to take into account the change of the inhomogeneity of the magnetic field near surface of the superconductor. For comparison in Fig.5 the curves dP/dH are performed for the superconductor with parameters , = 25,  $\Delta$  =  $1, \delta = 1$  for various angles  $\theta$ . It is clear seen here that with increasing of the angle the parameter A/B is sharply augmented. These significant changes of the NMR lineshape can essentially modify conclusions about the type of the vortex lattice. The results show that the inhomogeneity of the distribution of the magnetic field is appreciably changed for the different orientation of external field H with



Figure 4: A curve dP/dH for a case  $\theta = \pi/6$ . The horizontal line means dP/dH = 0, the designations on x-axis correspond Fig.1. The dotted line - mode bulk. The continuous line is designed in view of the surface effects ( $\delta = 1$ , mode skin ). $\Delta$  : a - 0.75, b - 1, c - 1.25, d - 1.75. a) "bulk" -A/B = 1.99, "skin" -A/B = 4.08, b) "bulk" -A/B = 2.15, "skin" -A/B = 4.54, c) "bulk". A/B = 2.16, "skin" -A/B = 4.92, d) "bulk" -A/B = 2.26, "skin" -A/B = 5.53.

respect to the boundary of the surface of the anisotropic superconductor. So, the parameters of the line can be vary in perceptible limits. Thus the NMR method is very sensitive to the features of the magnetic field distribution of the type II superconductor. The NMR probing in the inclined magnetic fields will allow to obtain more detailed information about the parameters of the superconductor (such as a parameter of anisotropy , ).



Figure 5: dP/dH for various  $\theta$  in view of the surface effects ( $\delta = 1$ ,  $\Delta = 1$ , mode "skin"):  $a - \theta = 0, b - \theta = \pi/12, c - \theta = \pi/6, d - \theta = \pi/4, e - \theta = \pi/3$  and  $\theta = 5\pi/12$ . The horizontal line means dP/dH = 0, the designations on it correspond Fig.1.

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